Three Generations and Fermion Chirality from Universal Bifurcations

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Abstract

The Standard Model (SM) fails to account for either the triplication of fermion families or chiral symmetry breaking in the electroweak sector. Here we show that both phenomena arise from the approach to chaos of quantum theory near the Fermi scale.

Key words: Standard Model, fermion generations, chiral symmetry breaking, Feigenbaum route to chaos, dissipative maps, period-doubling bifurcations.

1. Introduction

Iterated maps of the unit interval are generic models of dynamical systems in discrete time [7-8]. The standard representation of these models is based on first order difference equations having the form

$$x_{n+1} = f(x_n, \lambda) \tag{1}$$

where $n = 1, 2, \ldots$ is the iteration index and λ a control parameter. The dynamics expressed by (1) can be either *conservative* or *dissipative*. In the former case, the function (1) is monotonic and describes a one-to-one mapping, whereas the latter case is non-monotonic and describes a two-to-one mapping. Typical examples of dissipative systems include the *quadratic map* and *unimodal maps* [1-5]. In 1978, Feigenbaum has discovered that the onset of chaos in quadratic maps occurs through *period-doubling bifurcations* driven by changes of the control parameter [1-2]. It was later shown that the period-doubling transition to chaos with the same universal attributes develops in many multi-dimensional dissipative nonlinear systems [3-5] In particular, unimodal maps of the form [8-9]

$$f_{\lambda}(x) = \lambda f(x) \tag{2}$$

$$x \in [-1, 1] \ ; \ \ f(0) = 1 \ ; \ \ \lambda < 1 \tag{3}$$

exhibit the following behavior: for small values of λ has a single stable fixed point and all nearby points converge to it under iterations of (2). Ramping up λ to a critical value λ_1 makes the fixed point unstable and produces a new stable pair of points of period 2. Further increasing λ to another value (λ_2) bifurcates this cycle into a cycle of period 4. The bifurcation process continues with a sequence of cycles of period 2^j , $j \geq 3$, eventually leading to a Cantor set structure that attracts almost all the points of the interval $[-1, 1]$. On letting λ increase beyond an endpoint value λ_N, $N >> 1$, stable periodic orbits surface again and split in a similar way. In the new sequence, λ scans another series of critical values corresponding to cycles of period $3 \cdot 2^j$, $j = 0,1,2,...$ and so on [8-9].

2. Chaotic behavior of quantum theory near the Fermi scale

Quantum Mechanics contains many instances where unimodal functions of the type (2) show up. For example, a quantum wave packet initially centered at $x = 0$ [10]

$$\psi(x,0) = \exp(-\frac{x^2}{2}) \tag{3}$$

evolves in time according to

$$\psi(x,t)=\exp(-\frac{i\omega t}{2})\psi(x,0)=\exp(-\frac{i\omega t}{2})\exp(-\frac{x^2}{2}) \tag{4}$$

Another example is a wavefunction localized in x space as in [10]

$$\psi(x)=\frac{\sin K(x-x_0)}{\pi(x-x_0)}\ ,\ K>>1 \tag{5}$$

whose momentum space representation is

$$\varphi(k)=\frac{\exp(-ikx_0)}{\sqrt{2\pi}} \tag{6}$$

Gauge theory demands that physical description of (4) or (6) be independent of any arbitrary phase factor $\exp(-i\chi)$ that multiplies either one of them. With reference to (4), a global gauge transformation defined by the complex parameter $\lambda_c=\exp(-i\chi)$ assumes the form

$$\psi'(x,t)=\lambda_c\psi(x,t)=\exp(-i\chi)\psi(x,t) \tag{7}$$

Following a standard procedure, we pass from Lorentzian to Euclidean coordinates using the prescription

$$\chi=\omega t=\omega(-it_E)=-i\omega t_E \tag{8}$$

$$\chi_E=\omega t_E \tag{9}$$

The Euclidean control parameter in the small angle approximation may be presented as

$$\lambda_E=\exp(\omega t_E)\approx 1+\omega t_E\ ,\quad |\omega t_E|=\varepsilon<<1 \tag{10}$$

In the context of the *minimal fractal manifold* (MFM) [11], (10) highlights the connection between the Euclidean control parameter λ_E and the scale-dependent deviation from spacetime dimensionality $\varepsilon(\mu) = 4 - D(\mu) << 1$. One concludes from (10) that the behavior of (7) is controlled by a parameter that runs with the energy scale as in $\lambda_E = \lambda_E(\mu)$.

Taken together, all these observations indicate that (7) undergoes a period-doubling transition to chaos upon letting λ_E scan a continuous range of values. In particular, as shown in [6],

1) Gauge bosons develop from the 2^j bifurcation pattern, while fermions from the $3 \cdot 2^j$ pattern. *The first stage of the fermion sector* $(j = 0)$ *contains a triplication of fermion generations*, in full agreement with the experimental basis of the SM.

2) The neutrino-antineutrino branch developed at $j = 0$ displays an intrinsic chiral asymmetry upon applying the CPT operator, which entails that only one of the Left (L) and Right (R) states exist. *It follows that chiral symmetry breaking is rooted in the neutrino sector and stems from the transition to chaos driven by the global gauge transformation (7).*

Both findings are consistent with [11-12], where the SM is conjectured to represent a *self-contained multifractal set*, whose composition is constrained by the so-called "sum-of-squares" relationship. The violation of chiral symmetry in the neutrino sector falls in line with spacetime anisotropy induced by fractional differential and integral operators near or above the Fermi scale [13].

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